Jae-Eun Lim

Jaeeunl

Project 1

Final Report

September 26, 2016

24-370 Engineering Design I

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Project 1: The Astronaut's Coat Rack

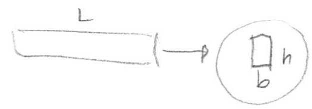
Goal: - as light as possible
- withstand 40 lb force for at least 10 seconds (<0.25 in deflection)

Rules: - rectangular region cannot be obscured
- only use up to 6 aluminum pegs (0.25 in)
- 3.5 in from right; 1.5 in from top

Material: - Evonik CYRO Acrylite FF (0.173 in thick; 12×6 in^2)

Failures to avoid: bending, torsion

Q: How to reduce bending stress? & deflection $\sigma = \frac{My}{I}$



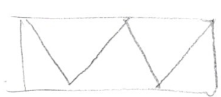
$I = \frac{1}{12}bh^3$

1. wide beam (increase h)
2. short length (decrease L)
3. use axial load (2 force-body) — no material wasted

Possible Models

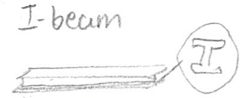
A. Features

Truss



Strut

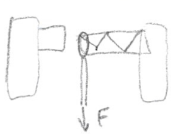
I-beam



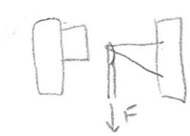
holes (to reduce mass)

Is it worth it?

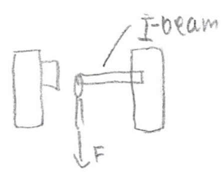
B. Rough models



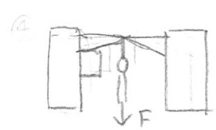
- may be heavy
- highly resistant to bending



- lighter
- resistant to bending



- hard to attach
- resistant to bending
- need to bond pieces



- heavier
- resistant to bending & torsion

C. Assumption

For simplicity ignore weight of bracket

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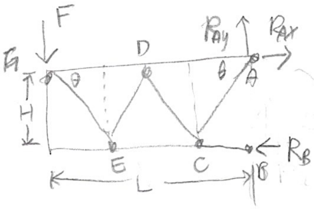


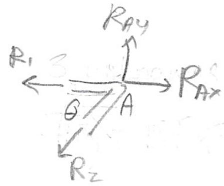
$\Sigma F_x = R_{Ax} - R_B = 0$

$\Rightarrow R_{Ax} = R_B$

$\Sigma F_y = R_{Ay} - F = 0 \Rightarrow R_{Ay} = F$

$\Sigma M|_A = FL - R_B H = 0$

$\Rightarrow R_B = \frac{FL}{H} = R_{Ax}$



$\Sigma F_y = R_{Ay} - R_2 \sin\theta = 0$

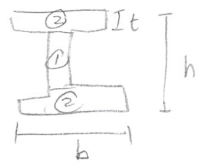
$\Rightarrow R_2 = \frac{R_{Ay}}{\sin\theta} = \frac{F}{\sin\theta}$

$\Sigma F_x = R_{Ax} - R_1 - R_2 \cos\theta = 0$

$\Rightarrow R_1 = \frac{-F}{\tan\theta} + \frac{FL}{H}$

$\Sigma F_y, \Sigma M$

$M = FL, \quad I = I_{neutral} + Ad^2$



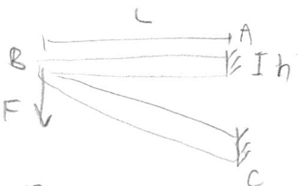
$I_1 = \frac{1}{12} b(h-2t)^3$

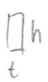
$I_2 = \frac{1}{12} bt^3 + bt\left(\frac{h}{2} - \frac{t}{2}\right)^3$

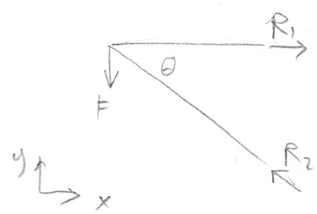
$I_{tot} = 2I_2 + I_1 = 2\left(\frac{1}{12}bt^3 + bt\left(\frac{h}{2} - \frac{t}{2}\right)^3\right) + \frac{1}{12} b(h-2t)^3$

Since $t \ll b, t \ll h \Rightarrow I_{tot} \approx \frac{1}{2} bth^2$

$\sigma = \frac{My}{I} = \frac{FL\frac{1}{2}h}{\frac{1}{2}bth^2} = \frac{FL}{bth}$ F, L, t given $\Rightarrow$ b, h has to be as large as possible





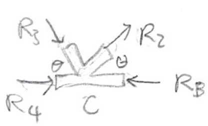


$\Sigma F_y = R_2 \sin\theta - F = 0 \Rightarrow R_2 = \frac{F}{\sin\theta}$

$\Sigma F_x = R_1 - R_2 \cos\theta \Rightarrow R_1 = R_2 \cos\theta = \frac{F}{\tan\theta}$

$\sigma_{AB} = \frac{F}{A} = \frac{F}{ht\tan\theta}, \quad \sigma_{BC} = \frac{F}{ht\sin\theta}$ F, t given $\Rightarrow$ increase h, $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

Continued

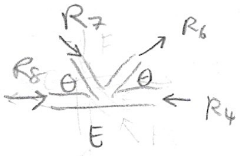


$\Sigma F_y = R_2 \sin\theta - R_3 \sin\theta = 0 \Rightarrow R_3 = R_2 = \frac{F}{\sin\theta}$

$\Sigma F_x = -R_B + R_4 + R_3\cos\theta + R_2\cos\theta = 0 \Rightarrow R_4 = \frac{-2F}{\tan\theta} + \frac{FL}{H}$

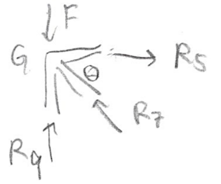
$\Sigma F_y = R_3 \sin\theta - R_6 \sin\theta = 0 \Rightarrow R_6 = R_3 = \frac{F}{\sin\theta}$

$\Sigma F_x = R_1 - R_5 - (R_6 + R_3)\cos\theta = 0 \Rightarrow R_5 = \frac{FL}{H} - \frac{F}{\tan\theta} - \frac{2F}{\tan\theta} = \frac{FL}{H} - \frac{3F}{\tan\theta}$



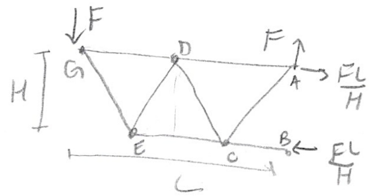
$$\Sigma F_y = R_6 \sin\theta - R_7 \sin\theta = 0 \Rightarrow R_7 = R_6 = \frac{F}{\sin\theta}$$

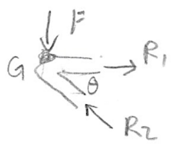
$$\Sigma F_x = (R_6 + R_7)\cos\theta - R_4 + R_8 = 0 \Rightarrow R_8 = -\frac{2F}{\tan\theta} + \frac{FL}{H} - \frac{2F}{\tan\theta} = \frac{FL}{H} - \frac{4F}{\tan\theta}$$



$$\Sigma F_y = R_7 \sin\theta + R_9 - F = 0 \Rightarrow R_9 = F - \frac{F}{\tan\theta}$$

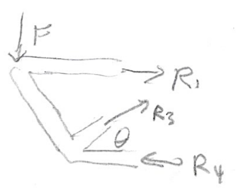
Get rid of 2 members (modification)





$$\Sigma F_y = R_2 \sin\theta - F = 0 \Rightarrow R_2 = \frac{F}{\sin\theta}$$

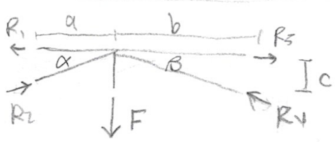
$$\Sigma F_x = R_1 - R_2\cos\theta = 0 \Rightarrow R_1 = \frac{F}{\tan\theta}$$



$$\Sigma F_y = R_3 \sin\theta - F = 0 \Rightarrow R_3 = \frac{F}{\sin\theta}$$

$$\Sigma F_x = R_1 + R_3\cos\theta - R_4 = 0 \Rightarrow R_4 = \frac{F}{\tan\theta} + \frac{F}{\tan\theta} = \frac{2F}{\tan\theta}$$

truss model has a member with greater stress than that of the simple strut model so strut model is better ⇒ eliminate truss model



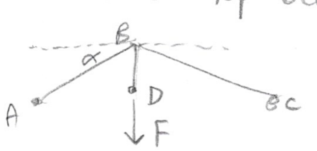
Assume $a \approx b$ so symmetric ⇒ $R_1 = R_3,\ R_2 = R_4,\ \alpha = \beta$

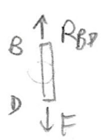
$$\Sigma F_y = 2R_2 \sin\alpha - F = 0 \Rightarrow R_2 = \frac{F}{2\sin\alpha}$$

$$\Sigma F_x = R_1 - R_1 + R_2\cos\alpha - R_2\cos\alpha = 0$$

⇒ top horizontal beam unnecessary

Get rid of top beam (modification)





$$\Sigma F_y = R_{BD} - F = 0 \Rightarrow R_{BD} = F \qquad \sigma_{BD} = \frac{R_{BD}}{A} = \frac{F}{th}$$

since F, t given increase h

cross-section

$$\sigma_{AB} \approx \sigma_{BC} \approx \frac{R_2}{A} = \frac{F}{2th\sin\alpha}$$

since F, t given increase h, $\alpha \to \frac{\pi}{2}$

but due to rectangular forbidden zone α has to be kept small

Using holes to reduce mass

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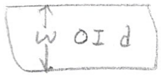
$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$



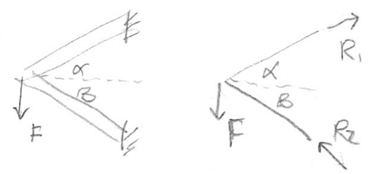
from stress concentration factor chart
as $\frac{d}{w} \to 1$ K_t decreases

$$FOS = \frac{S_y}{K_t \sigma_{nom}} = \frac{S_y}{\sigma_{max}}$$

Can test the effect of holes on Solidworks

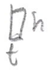
Alternative model

2-Force member



$$\Sigma F_y = R_1 \sin\alpha + R_2 \sin\beta - F = 0 \Rightarrow R_1 = \frac{F - R_2 \sin\beta}{\sin\alpha}$$

$$\Sigma F_x = R_1 \cos\alpha - R_2 \cos\beta = 0 \Rightarrow R_2 = \frac{F - R_2 \sin\beta}{\sin\alpha}\left(\frac{\cos\alpha}{\cos\beta}\right)$$



crosssection

$$R_2 = \frac{F/\cos\beta - R_2 \tan\beta}{\tan\alpha} \Rightarrow R_2(\tan\alpha + \tan\beta) = F/\cos\beta$$

$$\Rightarrow R_2 = \frac{F}{\cos\beta(\tan\alpha + \tan\beta)} \xRightarrow{\text{assume } \alpha \approx \beta} R_2 = \frac{F}{2\sin\alpha}$$

$$\Rightarrow R_1 = \frac{F - \frac{F\tan\beta}{\tan\alpha + \tan\beta}}{\sin\alpha} \xRightarrow{\text{assume } \alpha \approx \beta} R_1 = \frac{F}{2\sin\alpha}$$

$$\sigma = \frac{R}{A} = \frac{F}{2th\sin\alpha}$$

Since F, t given $\to$ increase h, $\alpha \to \frac{\pi}{2}$

Rough Draft

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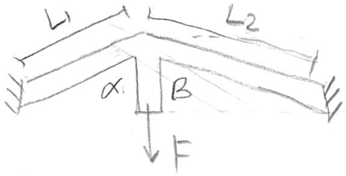
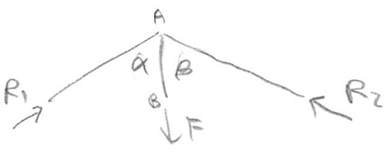
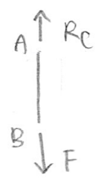


Load Analysis





$$\Sigma F_y = R_c - F = 0 \Rightarrow \boxed{R_c = F}$$ for center piece

tensile

$$\Sigma F_y = R_1\cos\alpha + R_2\cos\beta - F = 0$$

$$\Rightarrow R_1 = \frac{F}{\cos\alpha} - \frac{R_2\cos\beta}{\cos\alpha}$$

$$\Sigma F_x = R_1\sin\alpha - R_2\sin\beta = 0 \Rightarrow R_1 = \frac{R_2\sin\beta}{\sin\alpha}$$

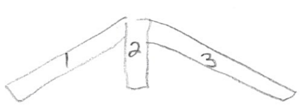
$$\Rightarrow \frac{F - R_2\cos\beta}{\cos\alpha} = \frac{R_2\sin\beta}{\sin\alpha} \Rightarrow \boxed{R_2 = \frac{F}{\cos\beta + \frac{\sin\beta}{\tan\alpha}}}$$

Compressive

$$R_1 = \frac{F\sin\beta}{\cos\beta\sin\alpha + \sin\beta\cos\alpha} = \boxed{\frac{F\sin\beta}{\sin(\alpha+\beta)} = R_1}$$

compressive

Failure Analysis



Beam 1: $\sigma_1 = \frac{R_1}{A} = \boxed{\frac{F\sin\beta}{th\sin(\alpha+\beta)} = \sigma_1}$ $\qquad FOS = \frac{\sigma_y}{\sigma_1}$

Beam 2: $\sigma_2 = \frac{R_c}{A} = \boxed{\frac{F}{th} = \sigma_2}$ $\qquad FOS = \frac{\sigma_y}{\sigma_2}$

Beam 3: $\sigma_3 = \frac{R_2}{A} = \boxed{\frac{F}{th\left(\cos\beta + \frac{\sin\beta}{\tan\alpha}\right)} = \sigma_3}$ $\qquad FOS = \frac{\sigma_y}{\sigma_3}$

Deflection: $\delta_1 = \sigma_1 \frac{L_1}{E} = \boxed{\frac{FL_1\sin\beta}{Eth\sin(\alpha+\beta)} = \delta_1}$ $\qquad \delta_2 = \sigma_2 \frac{L_1\cos\alpha}{E} = \boxed{\frac{FL_1\cos\alpha}{thE} = \delta_2}$

$$\delta_3 = \sigma_3 \frac{L_2}{E} = \boxed{\frac{FL_2}{Eth\left(\cos\beta + \frac{\sin\beta}{\tan\alpha}\right)} = \delta_3}$$

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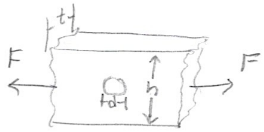
Holes



$\sigma_{nom} = \frac{F}{(D-d)t}$ $\sigma_{max} = K\sigma$

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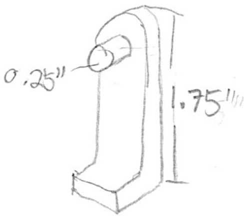
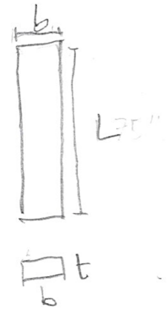
Suit Clip (6061-T6 Aluminum)

$\sigma_y = 4 \times 10^4$ psi, $E = 10^7$ psi

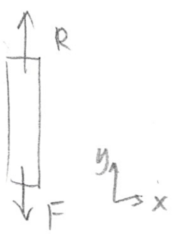


simplified model $\Longrightarrow$



$L = 1.75$ in
$b = 0.5$ in
$t = 0.15$ in

Load Analysis



$$\Sigma F_y = R - F = 0 \Rightarrow \boxed{R = F}$$

Failure Analysis

$$\sigma = \frac{R}{A} = \boxed{\frac{F}{bt} = \sigma} \qquad FOS = \frac{\sigma_y}{\sigma}$$

$$\delta = \sigma\frac{L}{E} = \boxed{\frac{FL}{btE} = \delta}$$

Inverse Analysis

$$FOS = \frac{\sigma_y bt}{F} = \frac{(4 \times 10^4 \text{ psi})(0.5 \text{ in})(0.15 \text{ in})}{40 \text{ lb}} = 75$$ very safe

$$\delta = \frac{FL}{btE} = \frac{(40 \text{ lb})(1.75 \text{ in})}{(0.5 \text{ in})(0.15 \text{ in})\, 10^7 \text{ psi}} = 9.3 \times 10^{-5} \text{ in}$$ very little deflection (negligible)

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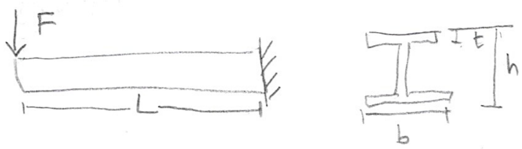
Model 1 - I-beam



* Not as good idea since need to use adhesive which may increase mass

Load Analysis

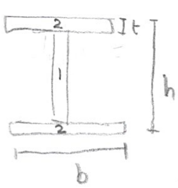
$\Sigma F_y = R - F = 0 \Rightarrow \boxed{R = F}$

$\Sigma F_x = FL - M = 0 \Rightarrow \boxed{M = FL}$

Failure Analysis

Deflection: $\delta_{max} = \dfrac{FL^3}{3EI}$

$FOS = \dfrac{\sigma_y}{\sigma_{max}}$ need to reduce σ_{max} as much as possible



$I_{tot} = I_1 + 2I_2 = \frac{1}{12} b(h-2t)^3 + 2\left[\frac{1}{12} bt^3 + bt\left(\frac{h}{2} - \frac{t}{2}\right)^3\right]$

since $t \ll b$ and $t \ll h$, simplify I_{tot}

$\boxed{I_{tot} \approx \frac{1}{2} bth^2}$

$\sigma_{max} = \dfrac{My}{I} = \dfrac{FL\frac{1}{2}h}{\frac{1}{2}bth^2} = \boxed{\dfrac{FL}{bth} \approx \sigma_{max}}$, $\boxed{\delta_{max} = \dfrac{2FL^3}{3Ebth^2}}$

Model 1 continued...

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Inverse Analysis

$F = 40\ \text{lb},\ L = 4\text{in},\ t = 0.173\ \text{in},\ \sigma_y = 5420\ \text{psi},\ E = 4\times10^5\ \text{psi},\ FOS_{desired} = 2$

$SG = 1.19$

$$FOS_d = \frac{\sigma_y}{\sigma_{max}} = \frac{\sigma_y b t h}{FL} \Rightarrow bh = \frac{FL\,FOS_d}{\sigma_y t} \Rightarrow b = \frac{FL\,FOS_d}{\sigma_y t h} \quad ①$$

Also make sure deflection $\delta_{max} \leq 0.25$ in $\rightarrow$ for safety let $\delta_{max} = 0.2$ in

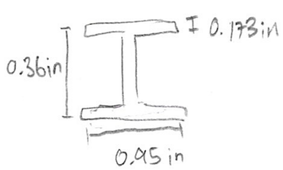
$$\delta_{max} = \frac{2FL^3}{3Ebth^2} \Rightarrow bh^2 = \frac{2FL^3}{3Et\delta_{max}} \Rightarrow b = \frac{2FL^3}{3Et\delta_{max}h^2} \quad ②$$

Set eqns ① and ② equal

$$\frac{\cancel{F}\cancel{L}\,FOS_d}{\sigma_y \cancel{t}\cancel{h}} = \frac{2\cancel{F}L^{\cancel{3}\,2}}{3E\cancel{t}\delta_{max}h^{\cancel{2}}} \Rightarrow h = \frac{2L^2\sigma_y}{FOS_d\,3E\,\delta_{max}}$$

$$h = \frac{2(4\text{in})^2(5420\ \text{psi})}{2(3)(4\cdot10^5\ \text{psi})(0.2\ \text{in})} = 0.36\ \text{in}$$

And from eqn ① $\quad b = \dfrac{(40\ \text{lb})(4\ \text{in})(2)}{(5420\ \text{psi})(0.173\text{in})(0.36\text{in})} = 0.95\ \text{in}$



Mass of acrylic needed

$\rho_a = SG \times \rho_{water},\quad \rho_w = 0.0361\ \text{lb/in}^3$

$V_{tot} = htL + 2(btL)$

$$m = \rho_a V_{tot} = SG\rho_w(htL + 2btL)$$
$$= (1.19)(0.0361\ \text{lb/in}^3)[(0.36\text{in})(0.173\text{in})(4\text{in}) + 2(0.95\text{in})(0.173\text{in})(4\text{in})]$$
$$= 0.0672\ \text{kg}$$

Model 2

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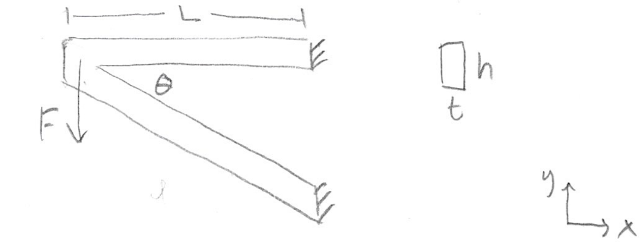
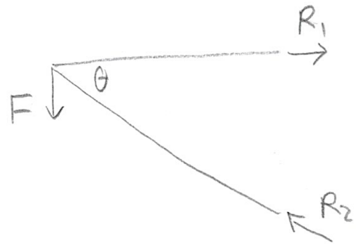


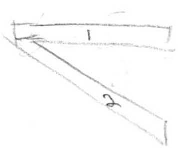
Load Analysis



$\Sigma F_y = R_2 \sin\theta - F = 0 \Rightarrow \boxed{R_2 = \frac{F}{\sin\theta}}$ compressive

$\Sigma F_x = R_1 - R_2 \cos\theta = 0 \Rightarrow \boxed{R_1 = \frac{F}{\tan\theta}}$ tensile

Failure Analysis



Beam 1: $\sigma_1 = \frac{R_1}{A} = \boxed{\frac{F}{th\tan\theta} = \sigma_1}$ $FoS = \frac{\sigma_y}{\sigma_1}$

Beam 2: $\sigma_2 = \frac{R_2}{A} = \boxed{\frac{F}{th\sin\theta} = \sigma_2}$ $FoS = \frac{\sigma_y}{\sigma_2}$

Deflection: $\delta_1 = \sigma_1 \frac{L}{E} = \boxed{\frac{FL}{Eth\tan\theta} = \delta_1}$

$\delta_2 = \sigma_2 \frac{L}{E\cos\theta} = \boxed{\frac{FL}{Eth\cos\theta\sin\theta} = \delta_2}$

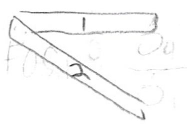
Model 2 continued...

Jae-Eun Lim

Inverse Analysis

$F = 40\text{ lb}$, $L = 4\text{ in}$, $t = 0.173\text{ in}$, $\sigma_y = 5420\text{ psi}$, $E = 4\times10^5\text{ psi}$, $FOS_{desired} = 2$, $SG = 1.19$

$\delta_{max} = 0.2\text{ in}$ (desired)



Beam 2: $FOS_d = \dfrac{\sigma_y}{\sigma_2} = \dfrac{\sigma_y t h \sin\theta}{F} \Rightarrow h = \dfrac{FOS_d \times F}{\sigma_y t \sin\theta}$ ①

$\delta_2 = \delta_{max} = \dfrac{FL}{Eth\sin\theta\cos\theta} \Rightarrow h = \dfrac{FL}{Et\delta_{max}\sin\theta\cos\theta}$ ②

Set eqns ① and ② equal

$$\frac{FOS_d \times \cancel{F}}{\sigma_y \cancel{t} \cancel{\sin\theta}} = \frac{\cancel{F}L}{E\cancel{t}\delta_{max}\cancel{\sin\theta}\cos\theta} \Rightarrow \theta = \cos^{-1}\left(\frac{L\sigma_y}{FOS_d\, E\, \delta_{max}}\right)$$

$$\theta = \cos^{-1}\left(\frac{(4\text{ in})(5420\text{ psi})}{2(4\cdot10^5\text{ psi})(0.2\text{ in})}\right) = 82.2°$$ (optimal but may be too large)

From eqn ① $h_2 = \dfrac{2(40\text{ lb})}{(5420\text{ psi})(0.173\text{in})\sin(82.2°)} = 0.086\text{ in}$ for Beam 2

Beam 1: $FOS_d = \dfrac{\sigma_y}{\sigma_T} = \dfrac{\sigma_y t h \tan\theta}{F} \Rightarrow h = \dfrac{FOS_d \times F}{\sigma_y t \tan\theta}$

$h_1 = \dfrac{2(40\text{ lb})}{(5420\text{ psi})(0.173\text{in})\tan(82.2°)} = 0.012\text{ in}$ for Beam 1

Mass of acrylic needed

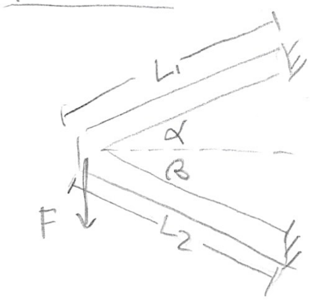
$\rho_a = SG \times \rho_w$, $\rho_w = 0.0362\text{ lb/in}^3$

$$V_{tot} = h_1 t L + \frac{h_2 t L}{\cos\theta} = tL\left(h_1 + \frac{h_2}{\cos\theta}\right)$$

$$m = \rho_a V_{tot} = SG\cdot\rho_w t L\left(h_1 + \frac{h_2}{\cos\theta}\right) = (1.19)(0.0362\text{ lb/in}^3)(0.173\text{in})(4\text{in})\left(0.012\text{in} + \frac{0.086\text{in}}{\cos 82.2°}\right) = 0.0192\text{ kg}$$

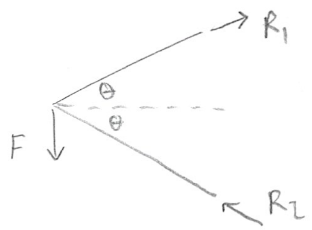
Jae-Eun Lim

Model 3



Load Analysis

Assumptions: $\alpha = \beta = \theta$, $\quad L_1 = L_2 = L$



$$\Sigma F_y = R_1 \sin\theta + R_2 \sin\theta - F = 0$$

$$\Rightarrow R_1 = \frac{F}{\sin\theta} - R_2$$

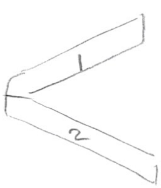
$$\Sigma F_x = R_1 \cos\theta - R_2 \cos\theta = 0 \Rightarrow R_1 = R_2$$

$$\Rightarrow R_1 = \frac{F}{\sin\theta} - R_1 \Rightarrow \boxed{R_1 = \frac{F}{2\sin\theta} = R_2}$$

R_1 is tensile

R_2 is compressive

Failure Analysis



Beam 1: $\sigma_1 = \frac{R_1}{A} = \boxed{\frac{F}{2th\sin\theta} = \sigma_1}$ $\quad$ $FOS = \frac{\sigma_y}{\sigma_1}$

Beam 2: $\sigma_2 = \frac{R_2}{A} = \boxed{\frac{F}{2th\sin\theta} = \sigma_2}$ $\quad$ $FOS = \frac{\sigma_y}{\sigma_2}$

Deflection: $\delta_1 = \sigma_1 \frac{L}{E} = \boxed{\frac{FL}{2thE\sin\theta} = \delta_1}$

$\delta_2 = \sigma_1 \frac{L}{E} = \boxed{\frac{FL}{2thE\sin\theta} = \delta_2}$

Model 3 continued... Jae-Eun Lim

Inverse Analysis

$F = 40\text{ lb}$, $SG = 1.19$, $t = 0.173\text{ in}$, $\sigma_y = 5420\text{ psi}$, $E = 4\times10^5\text{ psi}$, $FOS_{desired} = 2$

$\delta_{max} = 0.2\text{ in}$ (desired), $L\cos\theta = 4\text{ in} \Rightarrow L = \frac{4\text{ in}}{\cos\theta}$

Since beams 1 and 2 have stress and deflection of equal magnitude, let $\sigma_1 = \sigma_2 = \sigma$ and $\delta_1 = \delta_2 = \delta_{max}$

$$FOS_d = \frac{\sigma_y}{\sigma} = \frac{\sigma_y\, 2th\sin\theta}{F} \Rightarrow h\sin\theta = \frac{F\times FOS_d}{\sigma_y 2t} \quad ①$$

$$\delta_{max} = \frac{FL}{2thE\sin\theta} \Rightarrow h\sin\theta = \frac{FL}{\delta_{max}2tE} \quad ②$$

To balance h and $\sin\theta$, let $\theta = 45°$

Eqn ① $$h = \frac{(40\text{ lb})2}{(5420\text{ psi})2(0.173\text{ in})\sin(45°)} = 0.06\text{ in}$$

Eqn ② $$h = \frac{(40\text{ lb})(4\text{ in})/\cos(45°)}{(0.2\text{ in})(2)(0.173\text{ in})(4\cdot10^5\text{ psi})\sin(45°)} = 0.006\text{ in}$$

To be safe, choose $h = 0.06\text{ in}$

Mass of acrylic needed

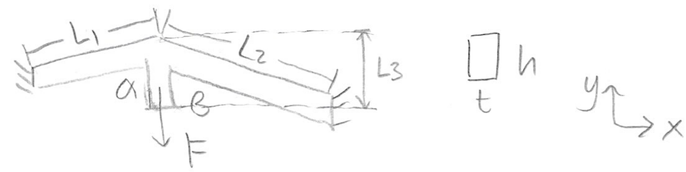
$\rho_a = SG\times\rho_w$; $\rho_w = 0.0362\text{ lb/in}^3$

$V_{tot} = 2thL$

$$m = \rho_a V_{tot} = SG\rho_w 2thL = (1.19)(0.0362\text{ lb/in}^3)(2)(0.173\text{ in})(0.06\text{ in})\left(\frac{4\text{ in}}{\cos 45°}\right) = 0.00506\text{ kg}$$

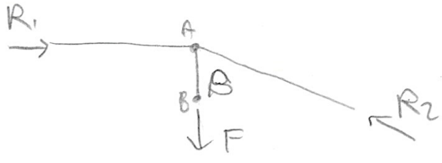
Model 4

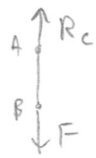
Jae-Eun Lim



Load Analysis

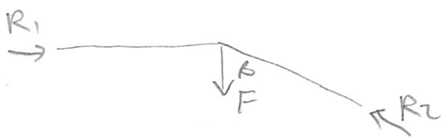
Assumption: $\alpha = 0°$





$\Sigma F_y = R_c - F = 0 \Rightarrow \boxed{R_c = F}$ for center beam

tensile



$\Sigma F_y = R_2 \cos\beta - F = 0 \Rightarrow \boxed{R_2 = \frac{F}{\cos\beta}}$ compressive

$\Sigma F_x = R_1 - R_2 \sin\beta = 0 \Rightarrow \boxed{R_1 = F\tan\beta}$ compressive

Failure Analysis

Beam 1: $\sigma_1 = \frac{R_1}{A} = \boxed{\frac{F\tan\beta}{th} = \sigma_1}$ $\quad FOS = \frac{\sigma_y}{\sigma_1}$

Beam 2: $\sigma_2 = \frac{R_c}{A} = \boxed{\frac{F}{th} = \sigma_2}$ $\quad FOS = \frac{\sigma_y}{\sigma_2}$

Beam 3: $\sigma_3 = \frac{R_2}{A} = \boxed{\frac{F}{th\cos\beta} = \sigma_3}$ $\quad FOS = \frac{\sigma_y}{\sigma_3}$

Deflection: $\delta_1 = \sigma_1 \frac{L_1}{E} = \boxed{\frac{FL_1\tan\beta}{thE} = \delta_1}$

$\delta_2 = \sigma_2 \frac{L_3}{E} = \boxed{\frac{FL_3}{thE} = \delta_2}$

$\delta_3 = \sigma_3 \frac{L_2}{E} = \boxed{\frac{FL_2}{thE\cos\beta} = \delta_3}$

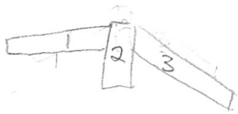
Model 4 continued...

Jae-Eun Lim

Inverse Analysis

$F = 40\text{ lb}$, $\alpha = 0°$, $t = 0.173\text{ in}$, $\sigma_y = 5420\text{ psi}$, $E = 4\times10^5\text{ psi}$, $FOS_{desired} = 2$

$\delta_{max} = 0.2\text{ in}$ (desired), $L_1 = 3\text{ in}$, $L_2 = \frac{4\text{ in}}{\cos\beta}$, $L_3 = 2\text{ in}$, $SG = 1.19$



Beam 2: $FOS_d = \frac{\sigma_y t h}{F} \Rightarrow h = \frac{F \times FOS_d}{\sigma_y t}$

$$h = \frac{(40\text{ lb})2}{(5420\text{ psi})(0.173\text{ in})} = 0.085\text{ in}$$

$$\delta_2 = \delta_{max} = \frac{FL_3}{thE} \Rightarrow h = \frac{FL_3}{t\delta_{max}E} = \frac{(40\text{ lb})(2\text{ in})}{(0.173\text{ in})(0.2\text{ in})(4\times10^5\text{ psi})} = 0.0058\text{ in}$$

To be safe, choose $h_2 = 0.085\text{ in}$ for Beam 2

Beam 1: $FOS_d = \frac{\sigma_y t h}{F\tan\beta} \Rightarrow \frac{h}{\tan\beta} = \frac{F\times FOS_d}{\sigma_y t}$ ①

$\delta_1 = \delta_{max} = \frac{FL_1\tan\beta}{thE} \Rightarrow \frac{h}{\tan\beta} = \frac{FL_1}{tE\delta_{max}}$ ②

To balance h and $\tan\beta$, let $\beta = 45°$

Eqn ① $h = \frac{(40\text{ lb})2\tan(45°)}{(5420\text{ psi})(0.173\text{ in})} = 0.085\text{ in}$

Eqn ② $h = \frac{(40\text{ lb})(3\text{ in})\tan(45°)}{(0.173\text{ in})(4\cdot10^5\text{ psi})(0.2\text{ in})} = 0.0087\text{ in}$

To be safe, let $h_1 = 0.085\text{ in}$ for Beam 1

Beam 3: $FOS_d = \frac{\sigma_y t h\cos\beta}{F} \Rightarrow h = \frac{F\times FOS_d}{\sigma_y t\cos\beta} = \frac{(40\text{ lb})(2)}{(5420\text{ psi})(0.173\text{ in})\cos45°} = 0.12\text{ in}$

$\delta_3 = \delta_{max} = \frac{FL_2}{thE\cos\beta} \Rightarrow h = \frac{FL_2}{tE\delta_{max}\cos\beta} = \frac{(40\text{ lb})(4\text{ in})/\cos45°}{(0.173\text{ in})(4\cdot10^5\text{ psi})(0.2\text{ in})\cos45°} = 0.023\text{ in}$

To be safe, choose $h_3 = 0.12\text{ in}$ for Beam 3

Jae-Eun Lim

Mass of acrylic needed

$\rho_a = SG \cdot \rho_w$, $\rho_w = 0.0361 \text{ lb/in}^3$

$V_{tot} = L_3 t h_3 + L_1 t h_1 + L_2 t h_2 = t(L_1 h_1 + L_2 h_2 + L_3 h_3)$

$m = \rho_a V_{tot} = SG \rho_w t (L_1 h_1 + L_2 h_2 + h_3 L_3)$

$= (1.19)(0.0361 \text{ lb/in}^3)(0.173\text{in})\left[(8\text{in})(0.085\text{in}) + \left(\frac{4\text{in}}{\cos 45°}\right)(0.085\text{in}) + (2\text{in})(0.12\text{in})\right]$

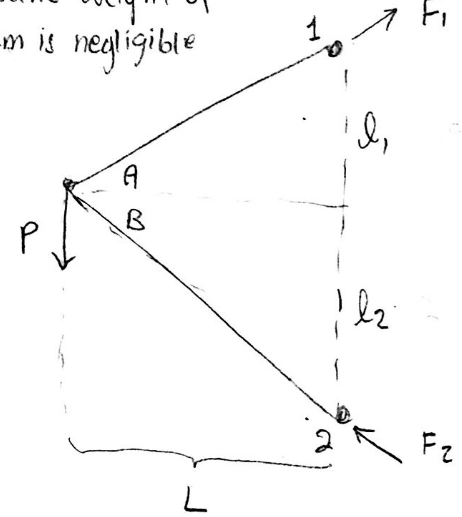
$= 0.0073$ kg

The lightest is Model 3, however need further analysis through Solidworks.

Additional Analyses after Initial Report

Load Analysis

Assume weight of beam is negligible



$$A = \tan^{-1}(\ell_1/L) \qquad B = \tan^{-1}(\ell_2/L)$$

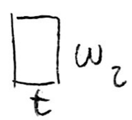
$$\Sigma M_1 = PL - F_2 \cos B\,(\ell_1+\ell_2) = 0$$

$$\Rightarrow F_2 = \frac{PL}{(\ell_1+\ell_2)\cos B} = \frac{PL}{(\ell_1+\ell_2)\cos\left(\tan^{-1}\left(\frac{\ell_2}{L}\right)\right)}$$

$$\Sigma M_2 = F_1 \sin A + F_2 \sin B - P = 0$$

$$\Rightarrow F_1 = \frac{P - F_2 \sin B}{\sin A} = \frac{P}{\sin A} - \frac{PL\tan B}{(\ell_1+\ell_2)\sin A}$$

$$= \frac{P}{\sin\left(\tan^{-1}\left(\frac{\ell_1}{L}\right)\right)} - \frac{PL\tan\left(\tan^{-1}(\ell_2/L)\right)}{(\ell_1+\ell_2)\sin\left(\tan^{-1}\left(\frac{\ell_1}{L}\right)\right)}$$



Failure Analysis

Tension/Compression

$$\sigma_{max} = \frac{F}{A} = \frac{F}{wt} \Rightarrow FOS = \frac{\sigma_y}{\sigma_{max}} = \frac{\sigma_y wt}{F} \Rightarrow w = \frac{FOS\cdot F}{\sigma_y t}$$

Top member: $$w_{1A} = \frac{FOS\cdot F_1}{\sigma_y t} = \frac{FOS}{\sigma_y t}\left(\frac{P}{\sin\left(\tan^{-1}\left(\frac{\ell_1}{L}\right)\right)} - \frac{P\ell_2}{(\ell_1+\ell_2)\sin\left(\tan^{-1}\left(\frac{\ell_1}{L}\right)\right)}\right)$$

Bottom member: $$w_{2A} = \frac{FOS\cdot F_2}{\sigma_y t} = \frac{FOS\cdot PL}{\sigma_y t\,(\ell_1+\ell_2)\cos\left(\tan^{-1}\left(\frac{\ell_2}{L}\right)\right)}$$

Buckling

$$F_{crit} = \frac{C\pi^2 EI}{\ell^2} = \frac{C\pi^2 E\,w t^3}{12\,\ell^2} \Rightarrow w = FOS \times \frac{F_{crit}\times 12\ell^2}{C\pi^2 E\,t^3}$$

Top member: $$w_{1B} = \frac{12\pi^2\ell^2 \times FOS \times P}{C\pi^2 E\,t^3 \sin\left(\tan^{-1}\left(\frac{\ell_1}{L}\right)\right)}\left(1 - \frac{\ell_2}{\ell_1+\ell_2}\right)$$

Bottom member: $$w_{2B} = \frac{12\,\ell_2^2 \times FOS \times PL}{C\pi^2 E\,t^3(\ell_1+\ell_2)\cos\left(\tan^{-1}(\ell_2/L)\right)}$$

Inverse Analysis

$P = 40\text{ lb},\ \sigma_y = 10^4\text{ psi},\ E = 400{,}000\text{ psi},\ C = 1,\ FOS = 1.65,\ \rho = 0.04\text{ lb/in}^3$

$t = 0.173\text{ in},\ L = 3.5\text{ in},\ l_1 = 1.5\text{ in},\ l_2 = 2\text{ in}$

$$W_{1A} = \frac{(1.65)}{(10^4\,psi)(0.173in)}\left(\frac{40\,lb}{\sin\left(\tan^{-1}\left(\frac{1.5in}{3.5in}\right)\right)} - \frac{(40\,lb)(2in)}{(1.5in+2in)\sin\left(\tan^{-1}\left(\frac{1.5in}{3.5in}\right)\right)}\right) = 0.044\text{ in}$$

$$W_{2A} = \frac{(1.65)(40\,lb)(3.5in)}{(10^4\,psi)(1.5+2\,in)(0.173in)\cos\left(\tan^{-1}\left(\frac{2in}{3.5in}\right)\right)} = 0.038\text{ in}$$

$$W_{1B} = \frac{12(1.5in)(1.65)(40\,lb)\left(1-\frac{2in}{1.5+2in}\right)}{(1)\pi^2(400\,000\,psi)(0.173in)^3\sin\left(\tan^{-1}\left(\frac{1.5in}{3.5in}\right)\right)} = 0.17\text{ in}$$

$$W_{2B} = \frac{12(2in)(1.65)(40\,lb)(3.5in)}{(1)\pi^2(400\,000\,psi)(0.173in)^3\cos\left(\tan^{-1}\left(\frac{2in}{3.5in}\right)\right)(1.5+2in)} = 0.22\text{ in}$$

Since $W_{1B} > W_{1A}$ and $W_{2B} > W_{2A}$ So let $W_1 = W_{1B}$, $W_2 = W_{2B}$

Now calculate mass:

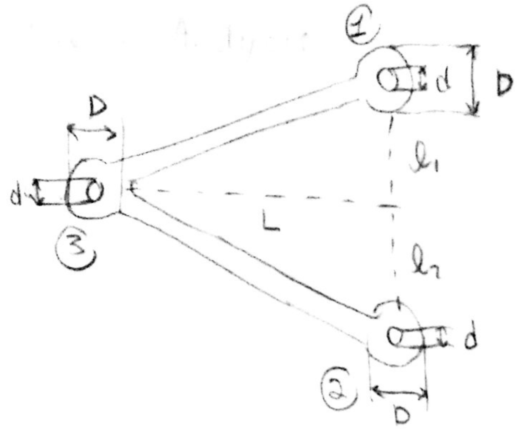
$$m = \rho V = \rho t\left(\frac{LW_1}{\cos A} + \frac{LW_2}{\cos B}\right) = \rho t L\left(\frac{W_1}{\cos(\tan^{-1}(l_1/L))} + \frac{W_2}{\cos(\tan^{-1}(l_2/L))}\right)$$

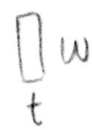
$$= (0.04\text{ lb/in}^3)(0.173in)(3.5in)\left(\frac{0.17in}{\cos\left(\tan^{-1}\left(\frac{1.5in}{3.5in}\right)\right)} + \frac{0.22in}{\cos\left(\tan^{-1}\left(\frac{2in}{3.5in}\right)\right)}\right)$$

$$= 0.011\text{ lb}$$

To optimize, iterate for different l_1, l_2, W_1, W_2 to find the set with lowest mass.

$$m = (0.011\,lb)\left(\frac{453.6g}{1\,lb}\right) = 4.99\text{ g}$$





$d = 0.25$ in

$D = 0.5$ in

Contact Stresses

①

$A_1 \approx dt$

$$\sigma_1 = \frac{F_1}{A_1} = \frac{PL}{dt\,(l_1 + l_2)\cos\left(\tan^{-1}\left(\frac{l_2}{L}\right)\right)}$$

②

$A_2 \approx dt$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{P}{dt\sin\left(\tan^{-1}\left(\frac{l_1}{L}\right)\right)}\left[1 - \frac{L\tan\left(\tan^{-1}\left(\frac{l_2}{L}\right)\right)}{l_1 + l_2}\right]$$

③

$A_3 \approx dt$

$$\sigma_3 = \frac{P}{A_3} = \frac{P}{dt}$$

$$FOS_1 = \frac{\sigma_y}{\sigma_1} = \frac{(10^4\,\text{psi})(0.25\,\text{in})(0.173\,\text{in})(1.5+2\,\text{in})\cos\left(\tan^{-1}\left(\frac{2\,\text{in}}{3.5\,\text{in}}\right)\right)}{(40\,\text{lb})(3.5\,\text{in})} = 9.39$$

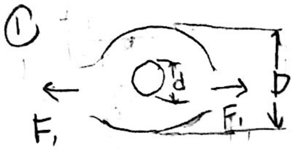
$$FOS_2 = \frac{\sigma_y}{\sigma_2} = \frac{(10^4\,\text{psi})(0.25\,\text{in})(0.173\,\text{in})\sin\left(\tan^{-1}\left(\frac{1.5\,\text{in}}{3.5\,\text{in}}\right)\right)}{(40\,\text{lb})\left(1 - \frac{(3.5\,\text{in})\tan\left(\tan^{-1}\left(2\,\text{in}/3.5\,\text{in}\right)\right)}{(1.5+2)\,\text{in}}\right)} = 9.94$$

$$FOS_3 = \frac{\sigma_y}{\sigma_3} = \frac{(10^4\,\text{psi})(0.25\,\text{in})(0.173\,\text{in})}{40\,\text{lb}} = 10.8$$

High FOS indicates safety. Contact stresses around the holes are not likely to cause failure except due to fatigue.

Stress Concentration

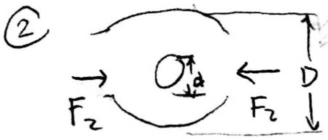
(1)



$$\frac{d}{D} = \frac{0.25}{0.5} = 0.5 \Rightarrow k_t \approx 2.2$$

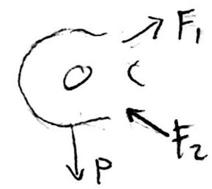
$$\sigma_{s1} = \frac{F_1}{A} = \frac{F_1}{(D-d)t} = \frac{PL}{(D-d)t(l_1+l_2)\cos\left(\tan^{-1}\left(\frac{l_2}{L}\right)\right)}$$

(2)

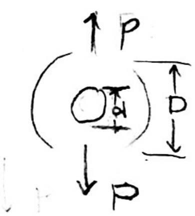


$$\sigma_{s2} = \frac{F_2}{A} = \frac{F_2}{(D-d)t} = \frac{P\left(1-\frac{l_2}{l_1+l_2}\right)}{(D-d)t\sin\left(\tan^{-1}\left(\frac{l_1}{L}\right)\right)}$$

(3)



simplify $\Rightarrow$



$$\sigma_{s3} = \frac{P}{A} = \frac{P}{(D-d)t}$$

$$\sigma_{max} = K_t \sigma_s \Rightarrow FOS = \frac{\sigma_y}{\sigma_{max}} = \frac{\sigma_y}{K_t \sigma_s}$$

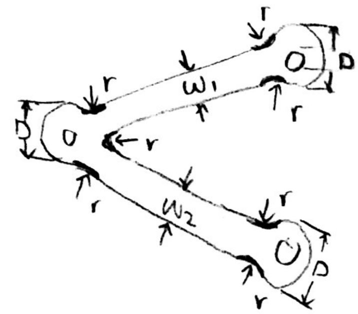
$$FOS_1 = \frac{\sigma_y}{K_t \sigma_{s1}} = \frac{(10^4)(0.25)(0.173)(3.5)\cos\left(\tan^{-1}\left(\frac{2.0}{3.5}\right)\right)}{(2.2)(40)(3.5)} = 4.27$$

$$FOS_2 = \frac{\sigma_y}{K_t \sigma_{s2}} = \frac{(10^4)(0.25)(0.173)\sin\left(\tan^{-1}\left(\frac{1.5}{3.5}\right)\right)}{(2.2)(40)\left(1-\frac{2}{3.5}\right)} = 4.52$$

$$FOS_3 = \frac{\sigma_y}{K_t \sigma_{s3}} = \frac{(10^4)(0.25)(0.173)}{(2.2)(40)} = 4.92$$

High FOS indicates safety. Stress concentrations around the holes are not likely to cause failure.

Reducing Stress Concentration using Fillets



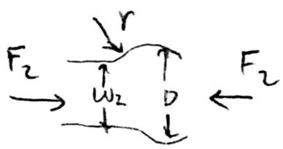
$r = 0.1\ in$

Top member: $\frac{r}{w_1} = \frac{0.1\ in}{0.17\ in} = 0.59$, $\frac{D}{w_1} = \frac{0.5}{0.17} = 2.94$

$\Rightarrow K_{tT} \approx 1.6$

$\sigma_{maxT} = K_{tT}\,\sigma_T = \frac{K_{tT} F_1}{w_1 t} \Rightarrow FOS_T = \frac{\sigma_y}{\sigma_{maxT}} = \frac{\sigma_y w_1 t}{K_{tT} F_1}$

Bottom member: $\frac{r}{w_2} = \frac{0.1\ in}{0.22\ in} = 0.45$, $\frac{D}{w_2} = \frac{0.5}{0.22} = 2.27$



$\Rightarrow K_{tB} \approx 1.6$

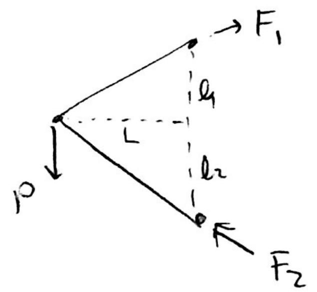
$\sigma_{maxB} = K_{tB}\,\sigma_B = \frac{K_{tB} F_2}{w_2 t} \Rightarrow FOS_B = \frac{\sigma_s}{\sigma_{maxB}} = \frac{\sigma_y w_2 t}{K_{tB} F_2}$

$FOS_T = \frac{(10^4)(0.17)(0.173)}{(1.6)(40)} = 4.59$

$FOS_B = \frac{(10^4)(0.22)(0.173)}{(1.6)(40)} = 5.95$

FOS are high enough for both top and bottom bars, to assume safety.

Displacement (Due to F_1 and F_2)



$$\delta_1 = \frac{F_1\sqrt{L^2+l_1^2}}{E w_1 t} = \frac{PL\sqrt{L^2+l_1^2}}{E w_1 t\,(l_1+l_2)\cos\left(\tan^{-1}\left(\frac{l_2}{L}\right)\right)}$$

$$\delta_2 = \frac{F_2\sqrt{L^2+l_2^2}}{E w_2 t} = \frac{P\sqrt{L^2+l_2^2}\left(1-\frac{l_2}{l_1+l_2}\right)}{E w_2 t\,\sin\left(\tan^{-1}\left(\frac{l_1}{L}\right)\right)}$$

$$\delta_1 = \frac{(40)(3.5)\sqrt{(3.5)^2+(1.5)^2}}{(4\cdot10^5)(0.17)(3.5)\cos\left(\tan^{-1}\left(\frac{2}{3.5}\right)\right)(0.173)} = 0.0149 \text{ in} \ll 0.25 \text{ in}$$

$$\delta_2 = \frac{(40)\sqrt{(3.5)^2+2^2}\left(1-\frac{2}{3.5}\right)}{(4\cdot10^5)(0.22)(0.173)\sin\left(\tan^{-1}\left(\frac{1.5}{3.5}\right)\right)} = 0.0115 \text{ in} \ll 0.25 \text{ in}$$

negligible deflection

Failure Load

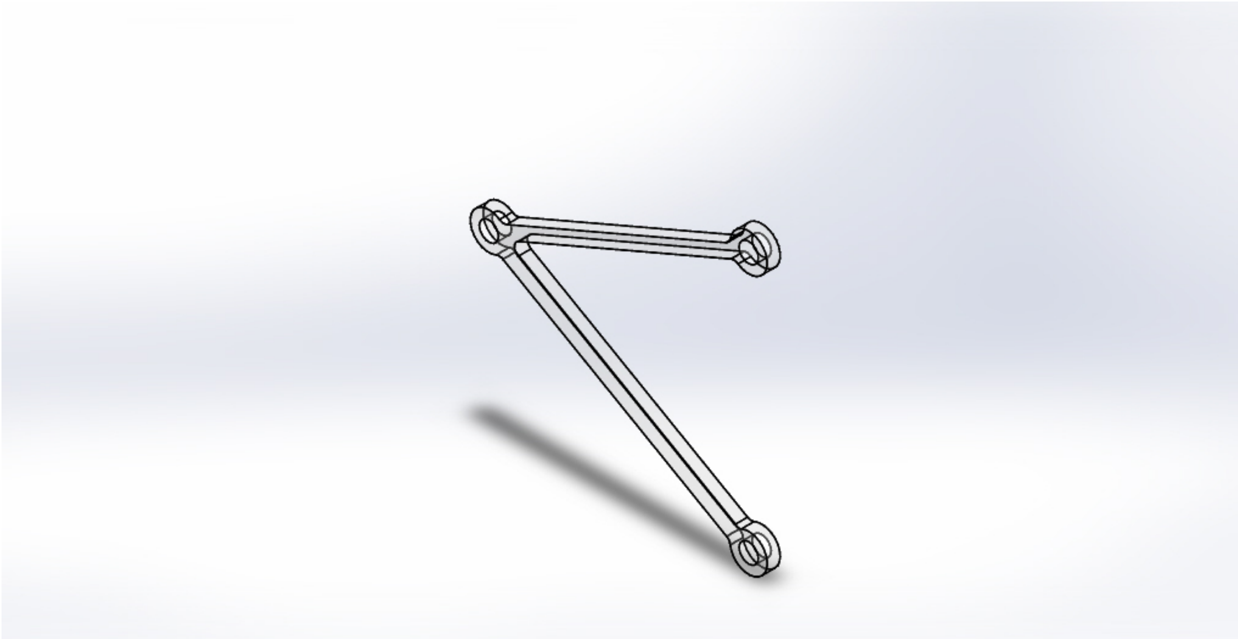
Top member:

$$F_{crit_T} = \frac{C\pi^2 E w_1 t^3}{12 l_1^2} = \frac{(1)\pi^2(4\cdot10^5)(0.17)(0.173)^3}{12(1.5)^2} = 128.7 \text{ lb}$$

Bottom member:

$$F_{crit_B} = \frac{C\pi^2 E w_2 t^3}{12 l_2^2} = \frac{(1)(\pi^2)(4\cdot10^5)(0.22)(0.173)^3}{12(2)^2} = 93.69 \text{ lb}$$

2. a.



2. b.
My bracket has a 0.17-inch wide top member and a 0.22-inch wide bottom member merging at an angle at the connection point of the suit clip. There are three 0.258-inch diameter holes for the suit clip and two pegs. The holes have 0.125-inch borders and the corners of the bracket have 0.1-inch fillets.

2. c.
My bracket theoretically has tensile stress in the top member and compressive in the bottom with no bending stresses. The extra 0.008-inch spaces in the holes reduce contact stresses. The 0.125-inch border around the holes and the 0.1-inch fillets in the corners reduce stress concentrations. Given the orientation of the members, the 0.17-inch width for the top member and 0.22-inch for the bottom member are the most optimal widths to carry forces with a factor of safety of 1.65, which is reasonable for reducing mass.

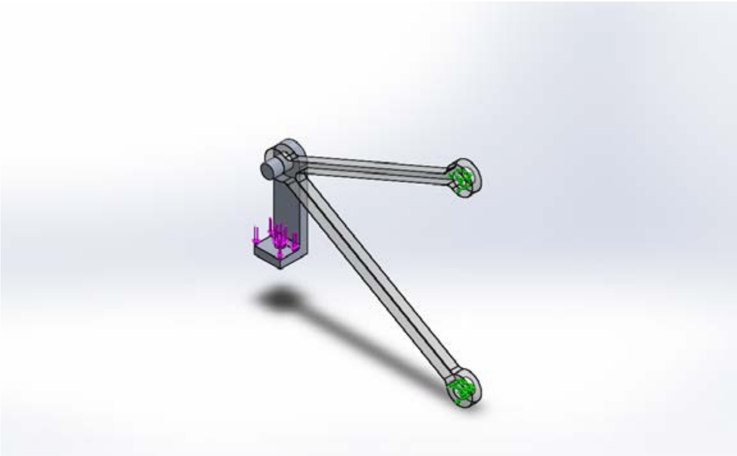
2. d.
m = 4.99 grams
Mass equals density times volume.

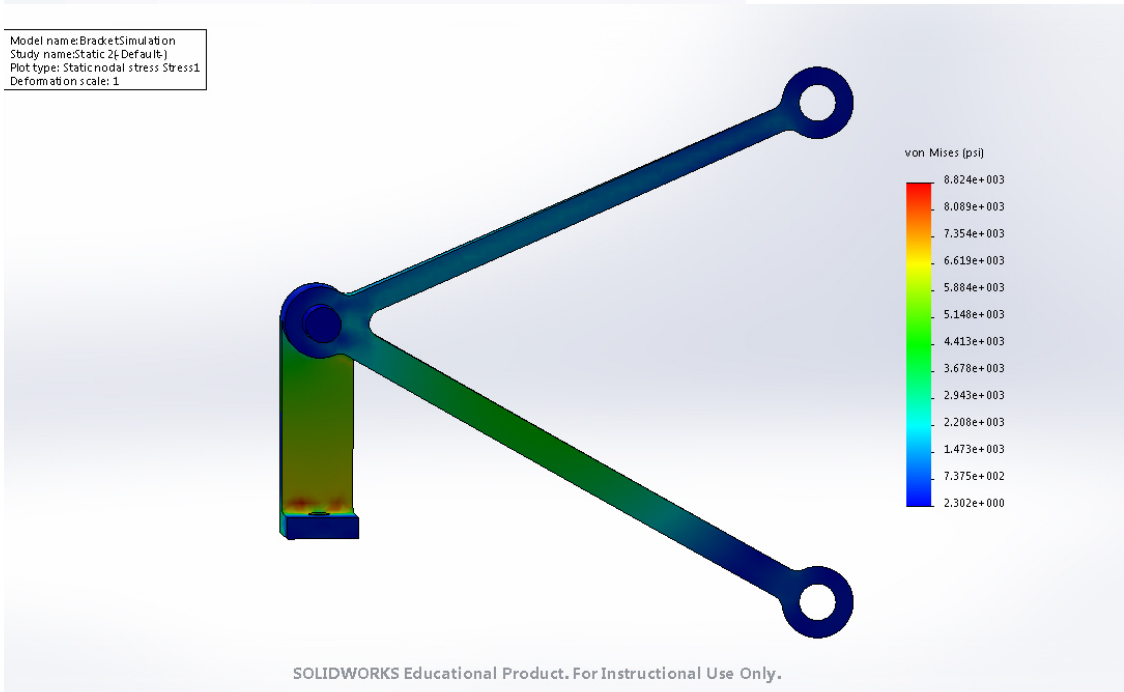
2. e.
FOS = 1.65
I made FOS a constraint. (pg 2)

2. f.
Fcrit_top_member = 128.7 lb
Fcrit_bottom_member = 93.69 lb
(pg 6)

2. g.
Buckling may occur due to compressive load in the bottom member.

4. b.

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Study name:Static 2(-Default-)
Plot type: Static nodal stress Stress1
Deformation scale: 1
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8.089e+003
7.354e+003
6.619e+003
5.884e+003
5.148e+003
4.413e+003
3.678e+003
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2.208e+003
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7.375e+002
2.302e+000
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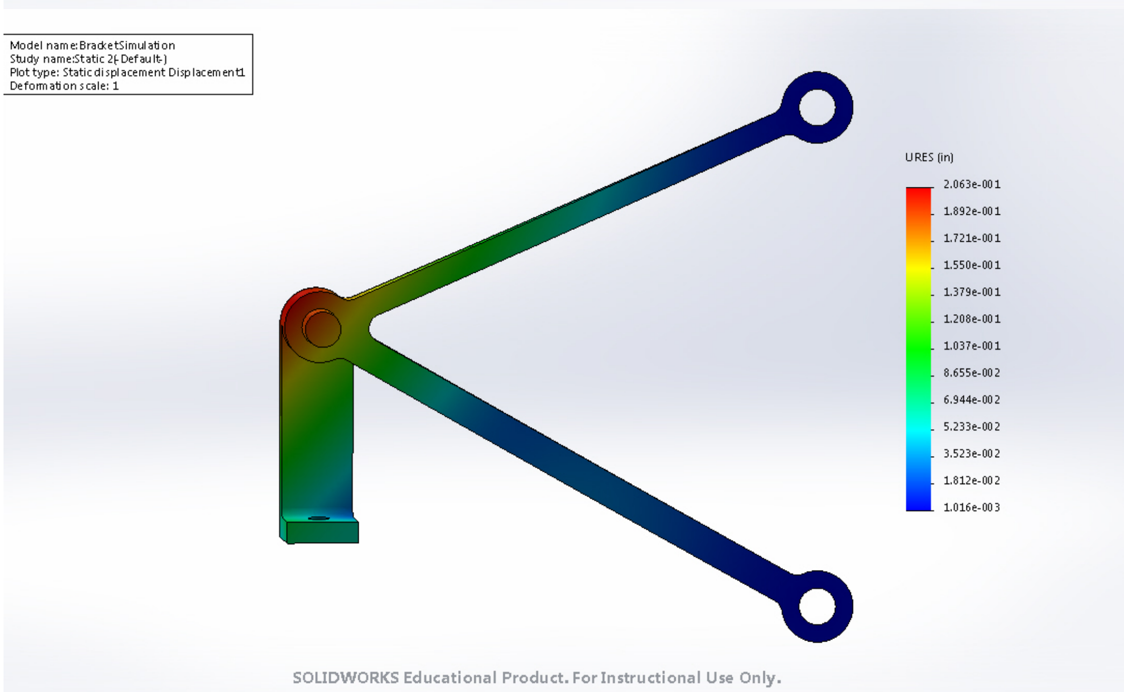
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Study name:Static 2(-Default-)
Plot type: Static displacement Displacement1
Deformation scale: 1
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1.037e-001
8.655e-002
6.944e-002
5.233e-002
3.523e-002
1.812e-002
1.016e-003
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5. b.

Since the tolerances of -0.005 +- 0.020 inches for outer dimensions and -0.007 +-0.020 inches for outer radii can reduce the widths of my bracket members, I rounded up the widths to the nearest hundredths. Also, since the tolerances of 0.002 +- 0.020 inches for inner diameters may increase the hole sizes, I made the holes 0.008 inches larger than the peg size so that there is just enough space to reduce contact stresses due to the peg.

6. b. Buckling Analysis